Inference and group statistics

Parts of material:
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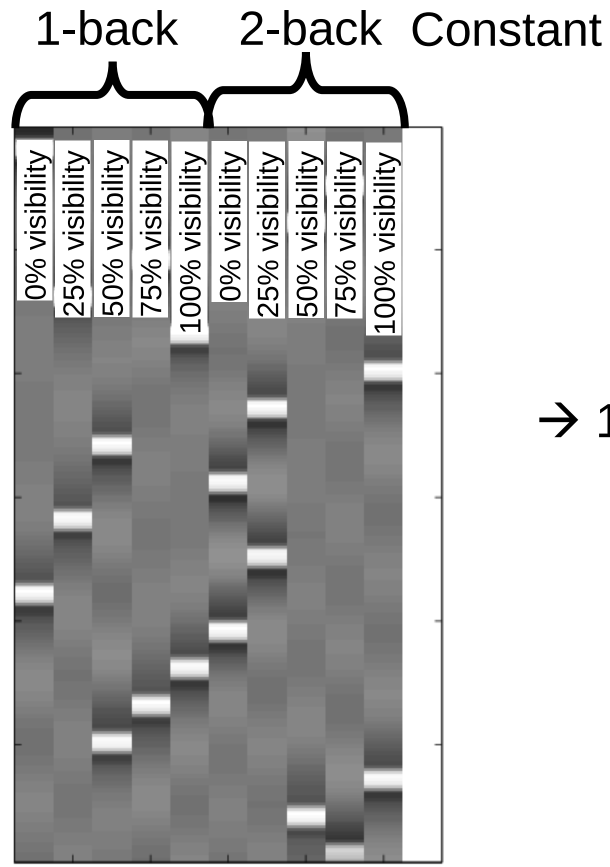
fMRI – example study



→ 10 beta maps

$$b_1 \ldots b_{10}$$

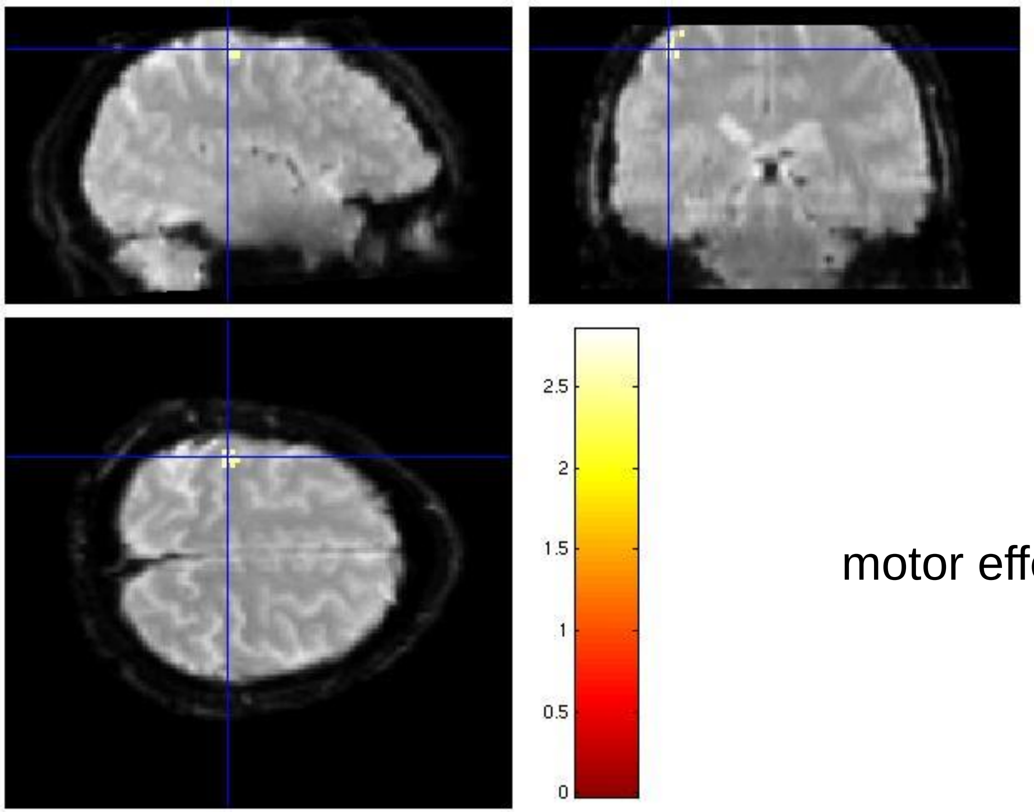
Contrasts:

Visibility	-2 -1 0 1 2 -2 -1 0 1 2
Cognitive load	-1 -1 -1 -1 -1 1 1 1 1 1
Interaction	-2 -1 0 1 2 2 1 0 -1 -2
Inverse interaction	2 1 0 -1 -2 -2 -1 0 1 2

→ various con maps

fMRI – parameter estimate maps

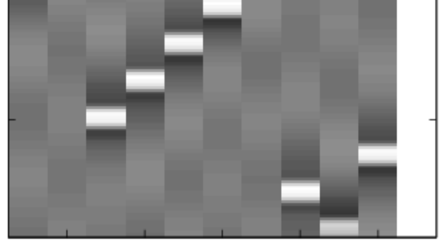
$$b_2 - b_1$$



motor effect

Is this significant?

Inference



Contrasts:

Visibility -2 -1 0 1 2 -2 -1 0 1 2

1. Is my beta/con value different from 0?

Null hypothesis H_0: beta/con = 0 (to be disproven)

Alternative hypothesis H_1: beta/con ≠ 0 (hopefully not disproven)

→ ***F test***

2. Is my beta/con value > 0?

Null hypothesis H_0: beta/con = 0 (to be disproven)

Alternative hypothesis H_1: beta/con > 0 (hopefully not disproven)

→ ***t test***

F test is undirected, t test is directed

Inference

Variance

- Deviation from a reference value
- E.g., deviation from mean μ of observations y_i

sum of squares

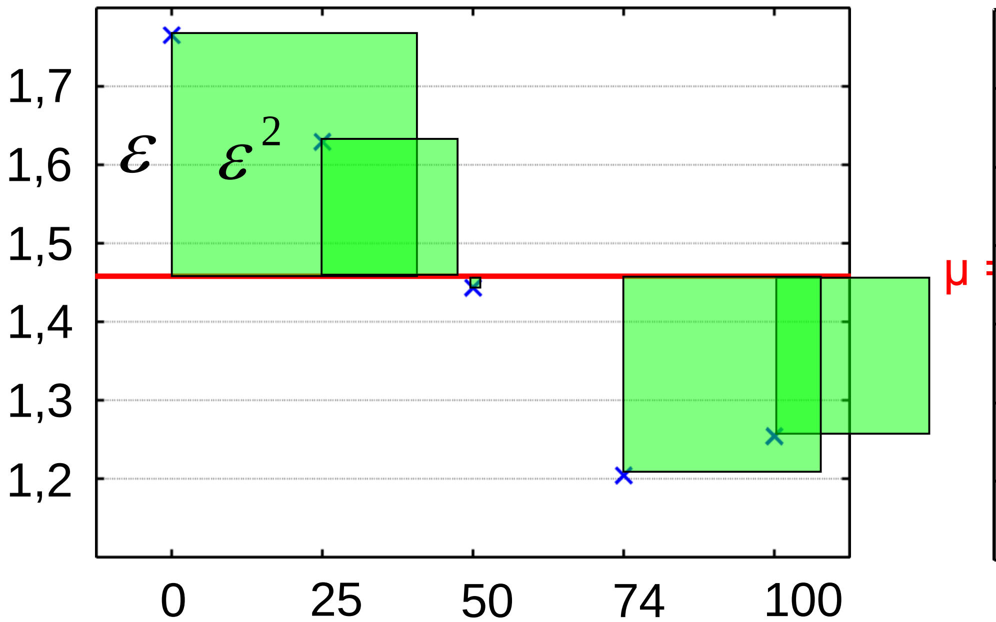
Variance: $$\hat{\sigma}^2 = \frac{\sum^{n} (y_i - \mu)^2}{n - 1}$$

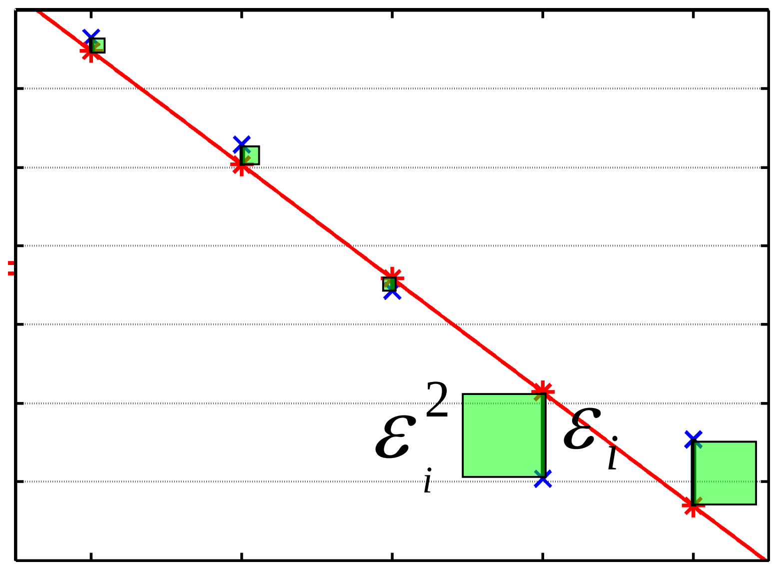
mean corrected: $$\hat{\sigma}^2 = \frac{\sum^{n} y_i^2}{n - 1}$$

Degrees of freedom: $n - 1$

Inference

Variance





$$\hat{\sigma}^2 = \frac{\sum^{n}(y_i - \mu)^2}{n - 1} = \frac{e_1^2 + \ldots + e_n^2}{n - 1}$$

$$y = b_0 1 + e \quad ; b_0 = n$$

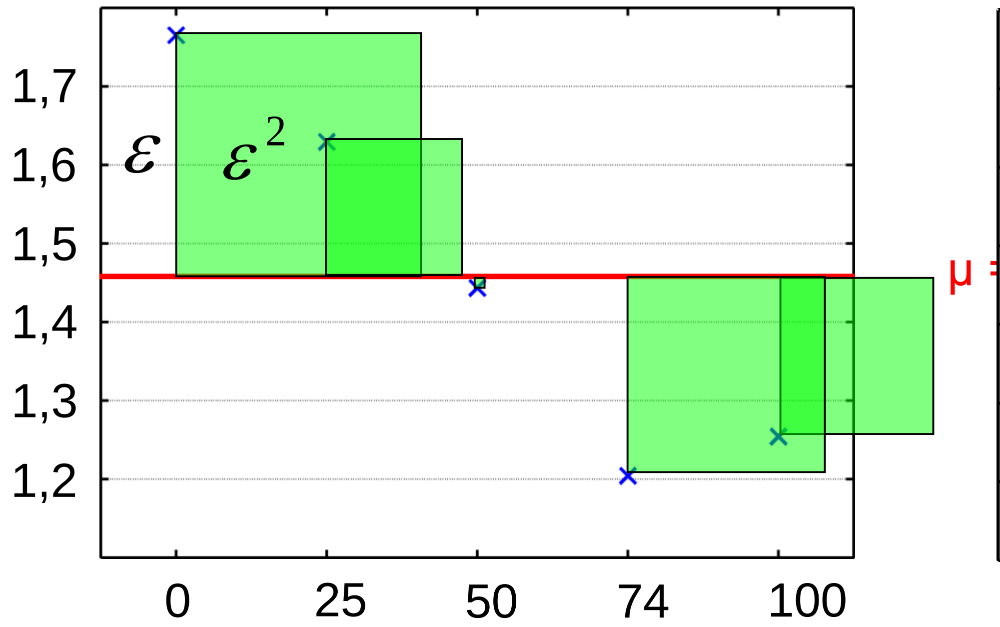
"reduced model“

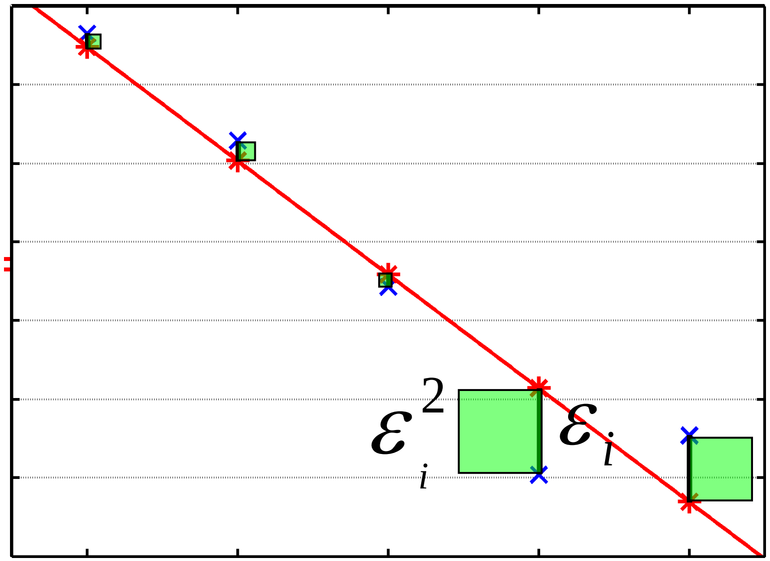
$$y = b_1 x_1 + b_0 1 + e$$

“full model“

Inference

Variance





$\hat{\sigma}^2$ variance of the full model (error variance)

$\hat{\sigma}^2 = \hat{\sigma}^2_{red}$ variance of the reduced model (total observed variance)

$\hat{\sigma}^2_{red} - \hat{\sigma}^2$ explained variance (experimental variance)

$$y = b_0 1 + e \quad ; b_0 = m$$

"reduced model"

$$y = b_1 x_1 + b_0 1 + e$$

"full model"

Inference

F test

How big is my explained variance relative to the error variance?

$$F = \frac{(\hat{\sigma}^2_{red} - \hat{\sigma}^2) / df1}{\hat{\sigma}^2 \; / \; df2}$$

explained variance (experimental variance)

variance of the full model (error variance)

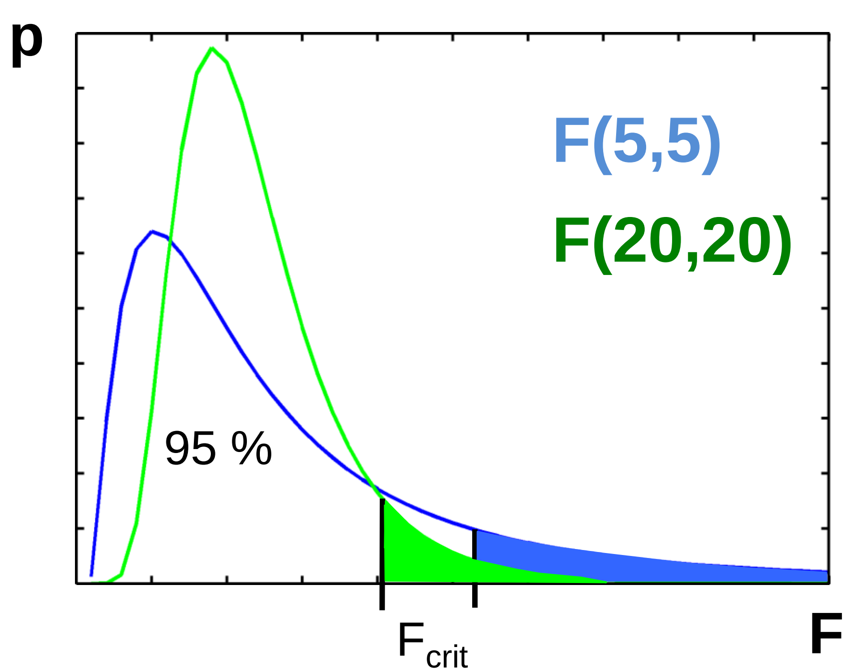
df1 = # experimental regressors
df2 = # data points - # all regressors

- More data points → F ↑
- More regressors → F ↓

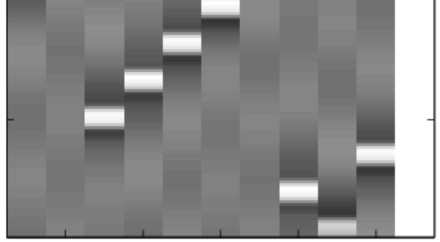
If F passes Fcrit
→ reject H_0, accept H_1

$$b_1 \neq 0$$

OR



Inference



Contrasts:

Visibility -2 -1 0 1 2 -2 -1 0 1 2

Contrast "visibility" ≠ 0

$$-2\beta_1 + (-1)\beta_2 + \ldots + 2\beta_{10} \neq 0$$

1. Is my beta/con value different from 0?

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→ ***F test***

2. Is my beta/con value > 0?

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→ ***t test***

F test is undirected, t test is directed

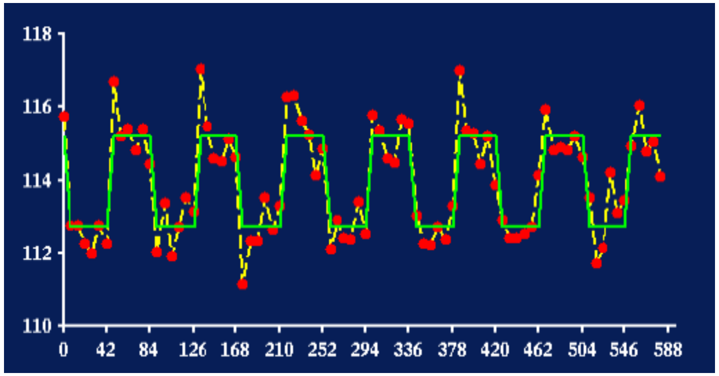
Inference

t test

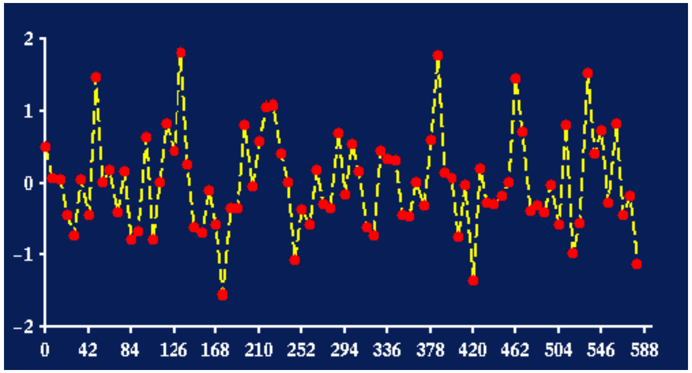
How big is my effect relative to the error variance?

$$t = \frac{\text{beta or con}}{\sqrt{\hat{\sigma}^2}}$$

$\hat{\sigma}^2$



Data y (signal time course) and fitted response $b_1 x_1$



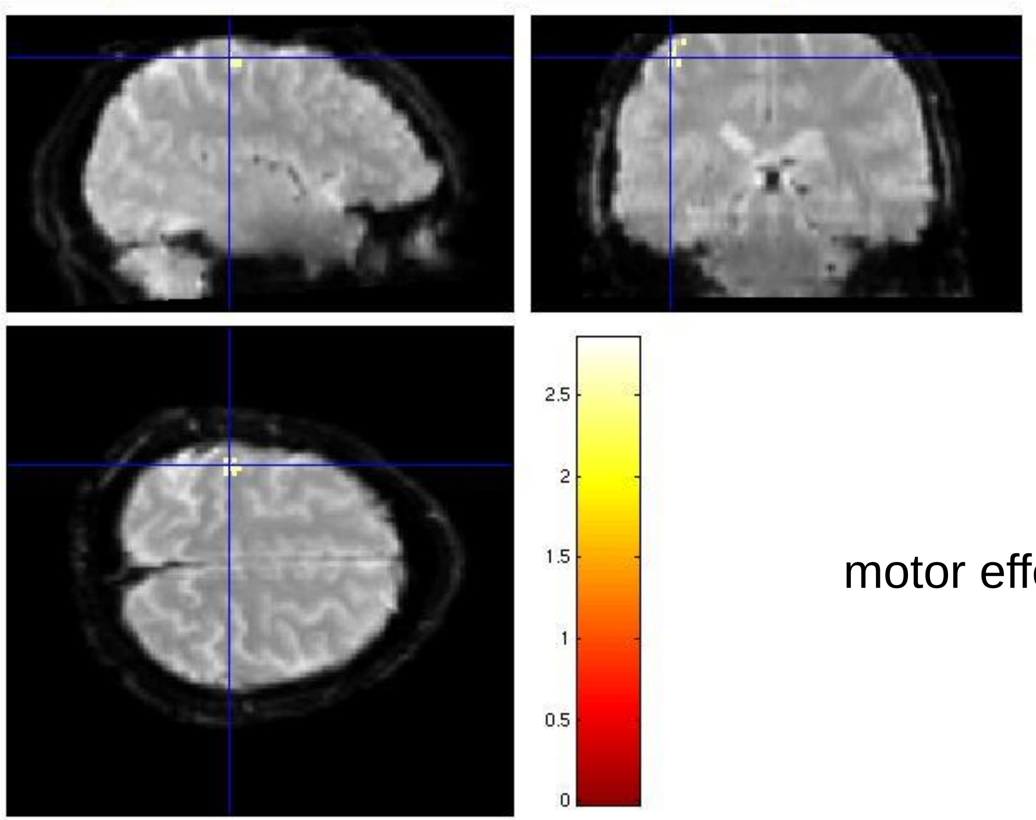
Data y – fitted response = residuals

$$y = b_1 x_1 + b_0 x_0 + e$$

$$y - b_1 x_1 = b_0 x_0 + e$$

F and t maps

E.g.,
for contrast
$b_2 - b_1$

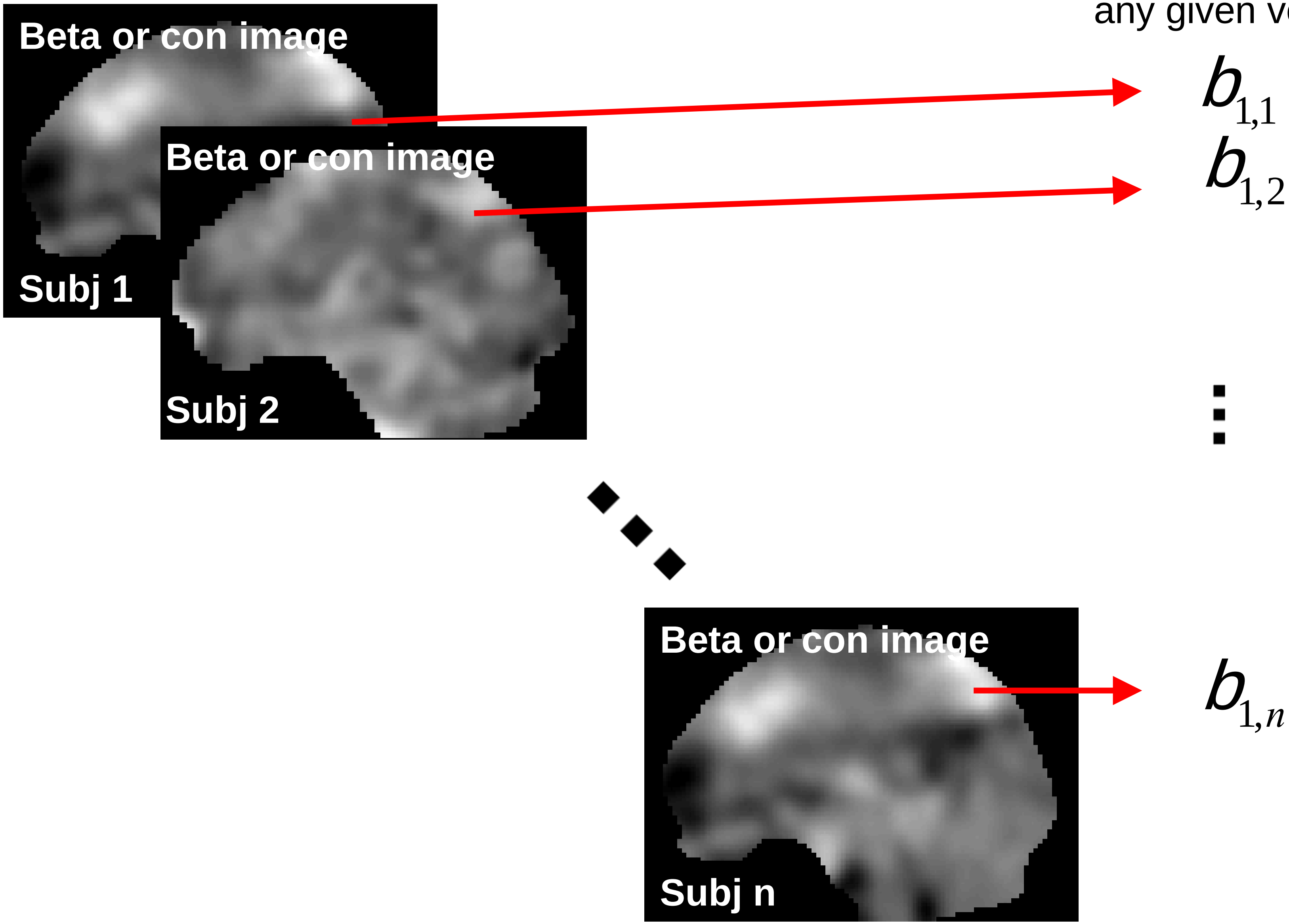


motor effect

Is this significant in my group?
→ “2nd level“

Group-level inference

E.g., beta1 maps: any given voxel:



Group-level inference

$b_{1,1}$
$b_{1,2}$
⋮
$b_{1,n}$

$$t = \frac{\text{beta or con}}{\sqrt{\hat{\sigma}^2}} = \frac{\bar{b}_1}{\sqrt{\hat{\sigma}^2}}$$

Single subject (1st level):
- many data points (observations) = signal time course
- variance within subject
  (deviation of time course from fitted regressors(s))
- many dfs

Group level (2nd level):
- few data points (observations) = subjects
- variance = deviation between subjects
  (random effects, RFX)
- few dfs

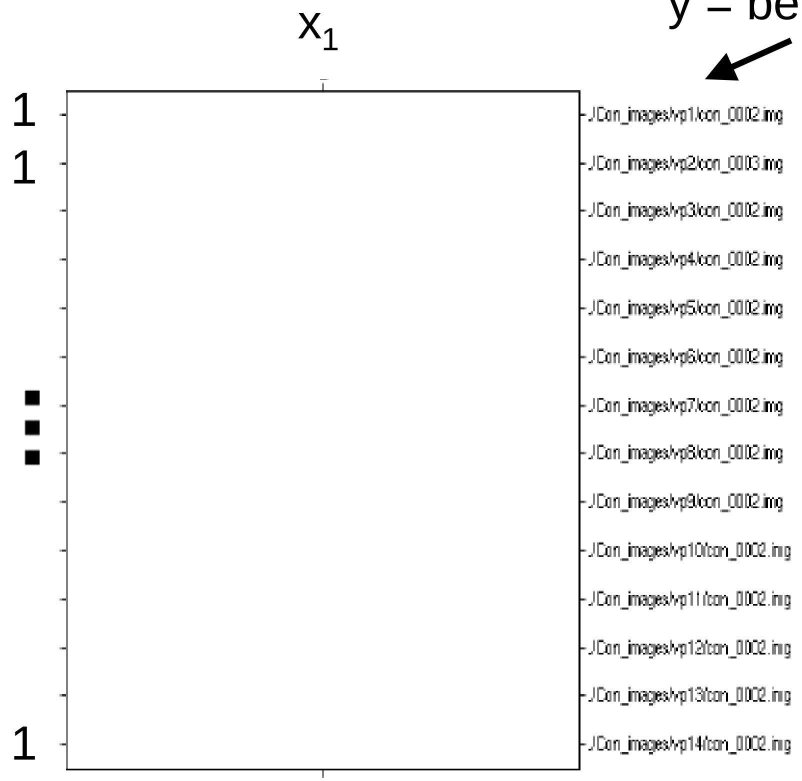
Group-level models (design matrices)

One-sample t test

Take any beta or con images from single-subject analysis

H$_1$: $\overline{b}_1 > 0$

y = beta/con images of all subjects



Group-level models (design matrices)

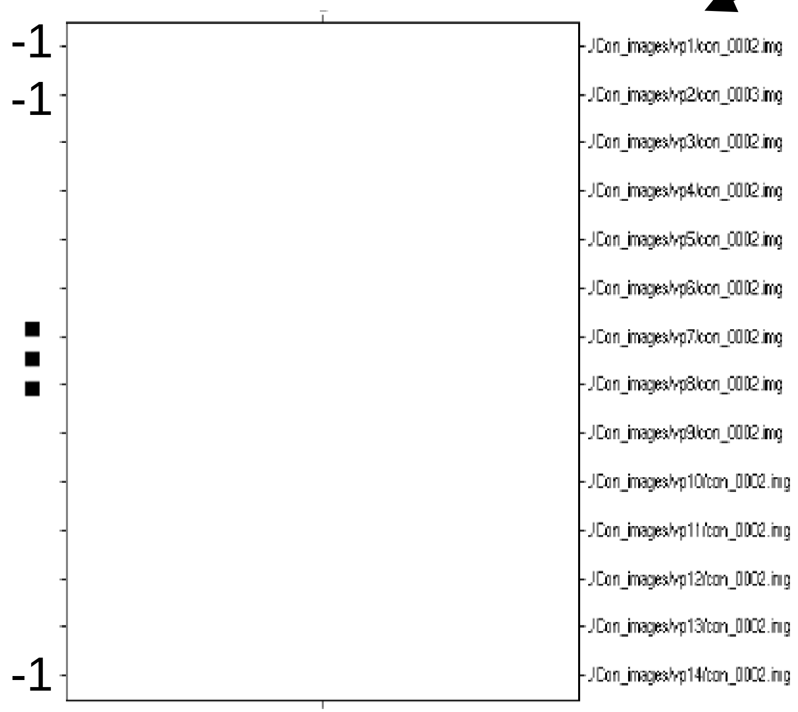
One-sample t test

Take any beta or con images from single-subject analysis

H$_1$: $\overline{b}_1 < 0$

y = beta/con images of all subjects

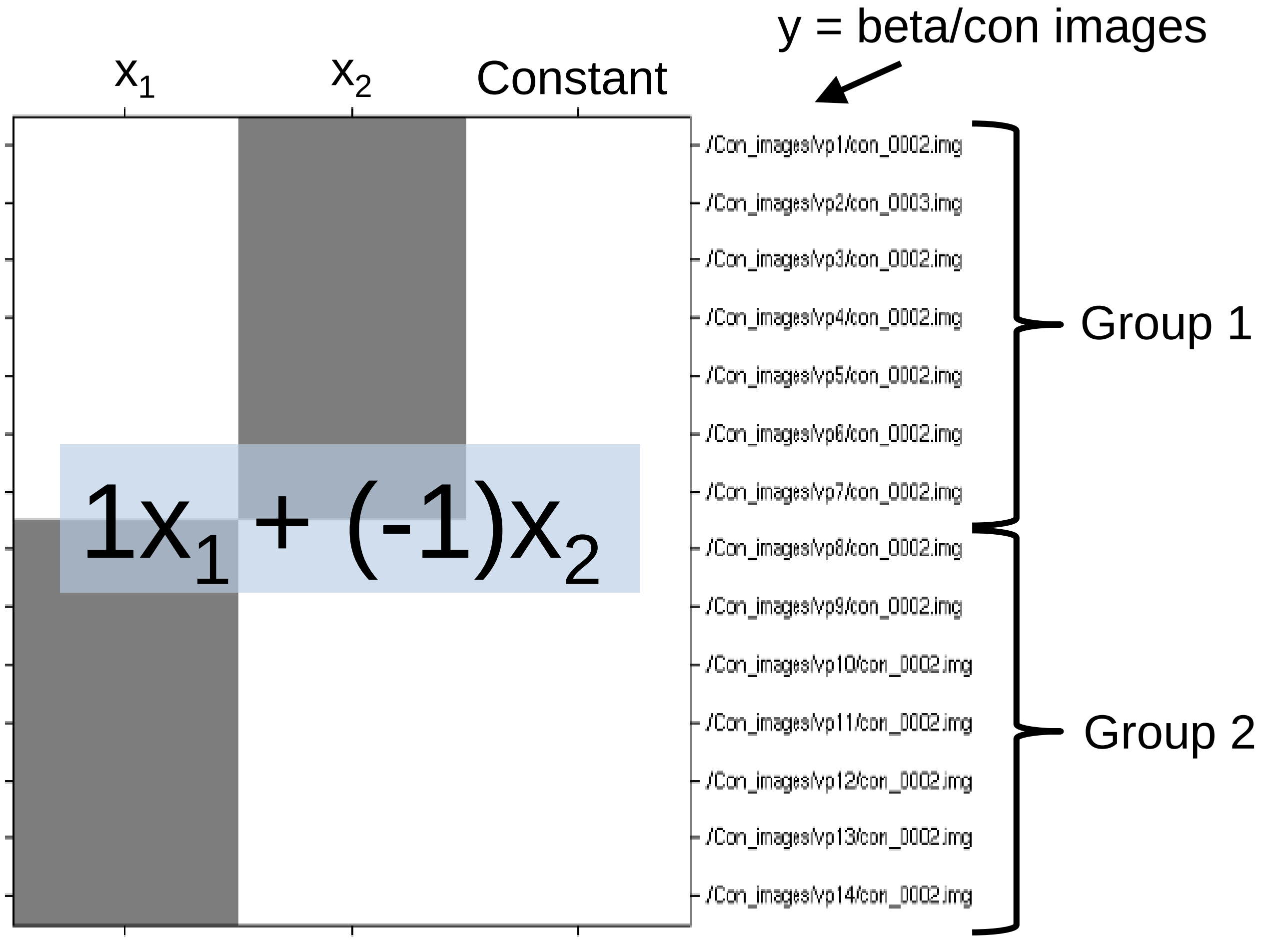
x_1



Group-level models (design matrices)

Two-sample t test

H_1: Group 1 > Group 2



x_1 = 1 1 1 1 1 1 1 0 0 0 0 0 0 0; x_2 = 0 0 0 0 0 0 0 1 1 1 1 1 1 1

Group-level models (design matrices)

Two-sample t test with covariate

H_1: Group 1 > Group 2