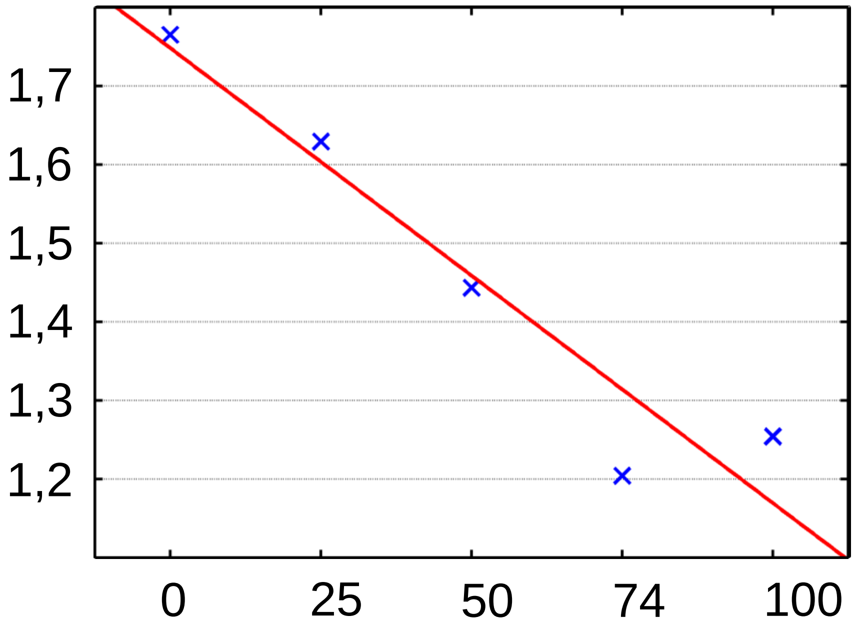
# General Linear Model

*Parts of material:*
courtesy of Tobias Sommer-Blöchl
Institute for Systems Neuroscience
University Medical Center Hamburg-Eppendorf (UKE)

## Basics

- GLM: (multiple) regression $y = ax + b + Error$
- y: dependent variable (Kriteriumsvariable)
- x: independent variable, regressor, “model“ (Prädiktorvariable)

| x | y |
|---|---|
| 0 | 1,77 |
| 25 | 1,63 |
| 50 | 1,44 |
| 75 | 1,20 |
| 100 | 1,25 |




⇒ **describe the relationship between x and y**

- Multiple regressors $x_j$ $y = a_1x_1 + a_2x_2 + \dots + a_mx_m + 1b + e$

⇒ **y: weighted linear combination of regressors $x_j$**

# Matrix notation

$$y = a_1 x_1 + a_2 x_2 + \ldots + a_m x_m + 1b + \varepsilon$$

Weighted regressors

Constant

Depend. variable

Error

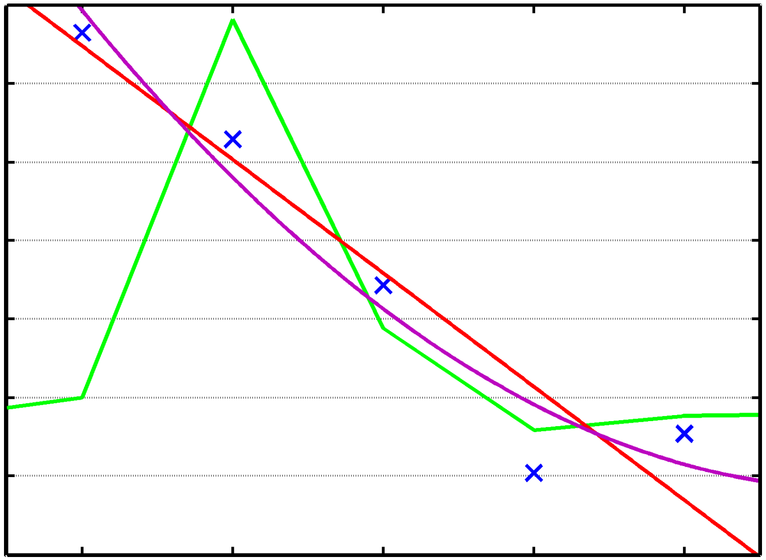
$$\begin{bmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{bmatrix} = \beta_1 \begin{bmatrix} x_{1,1} \\ \vdots \\ x_{i,1} \\ \vdots \\ x_{n,1} \end{bmatrix} + \beta_2 \begin{bmatrix} x_{1,2} \\ \vdots \\ x_{i,2} \\ \vdots \\ x_{n,2} \end{bmatrix} + \ldots + \beta_m \begin{bmatrix} x_{1m} \\ \vdots \\ x_{im} \\ \vdots \\ x_{nm} \end{bmatrix} + \beta_0 \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_i \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} & x_{10} \\ \vdots & \vdots & & \vdots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{im} & x_{i0} \\ \vdots & \vdots & & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} & x_{n0} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \\ \beta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_i \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

$$y = X\beta + \varepsilon$$

Scalar product
(Kreuzprodukt)

# Choosing a model



- What is the hypothesis about the x-y relationship?
- Which regressors are to be included in the model?

# Choosing a model – example: linear relationship

$$y = X\beta + \varepsilon$$

$x_1$ : Visibility

| y | x |
|---|---|
| 1,77 | 0 |
| 1,63 | 25 |
| 1,44 | 50 |
| 1,20 | 75 |
| 1,25 | 100 |

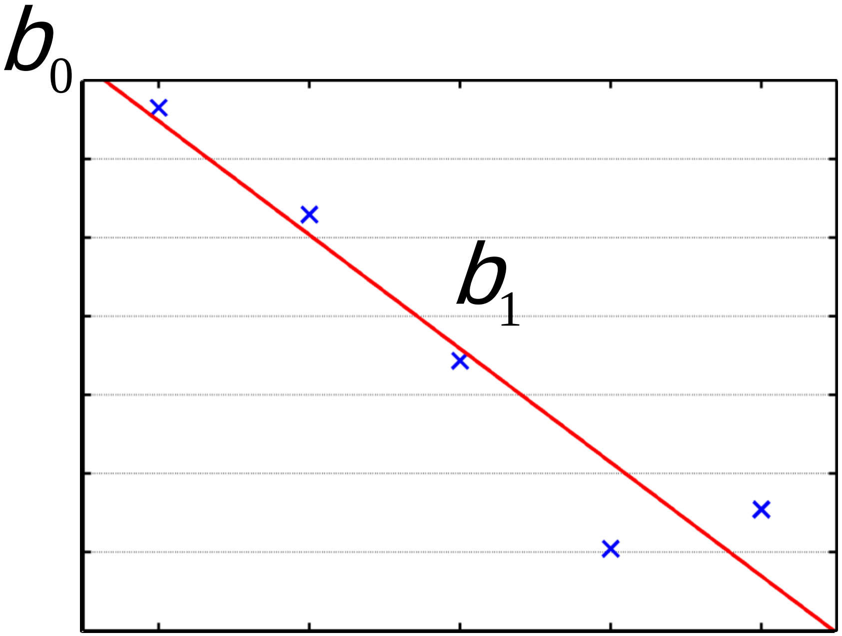
Reaction times

$$\begin{bmatrix} 1,77 \\ 1,63 \\ 1,44 \\ 1,20 \\ 1,25 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 25 & 1 \\ 50 & 1 \\ 75 & 1 \\ 100 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_i \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

$\beta_0$ : "Y-Achsenabschnitt"

$\beta_1$ : Slope (Steigung)

# Choosing a model – example: linear relationship

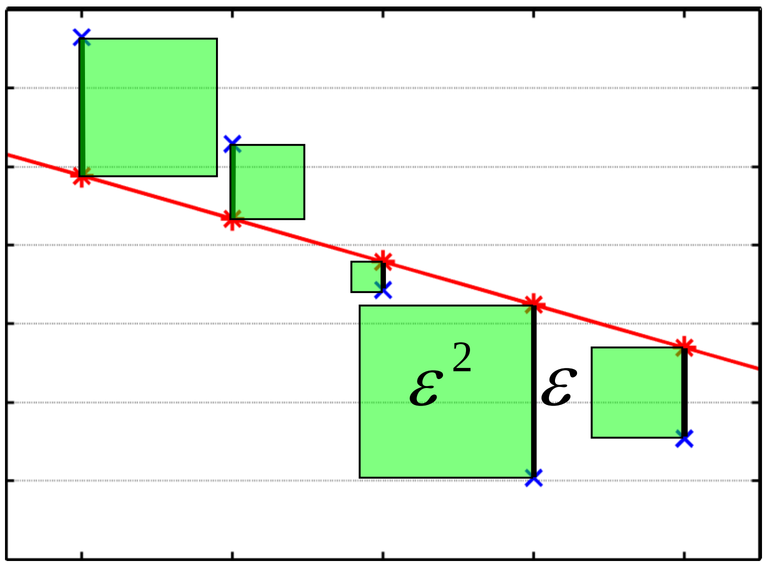


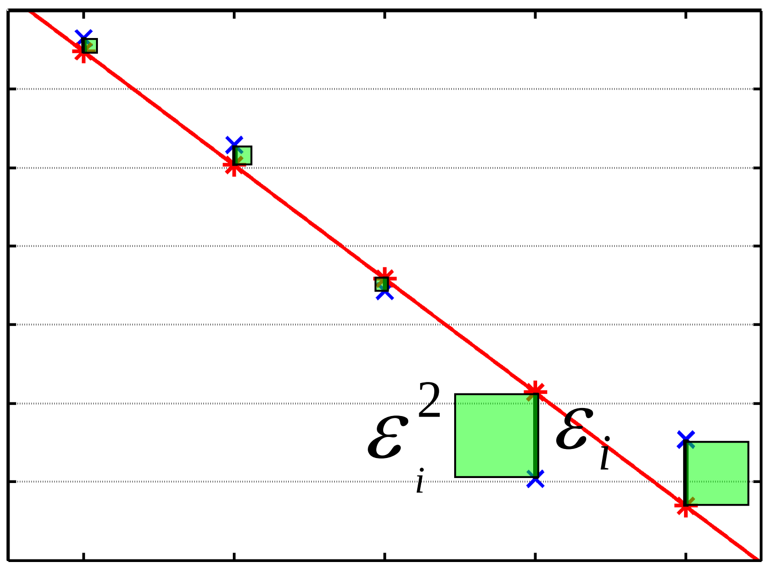
# Parameter estimation

- Get the “best“ betas
- Minimal error

$\Rightarrow$ **Criterion of least sum of squares (LSS)**

(minimale Summe der Abweichungsquadrate)

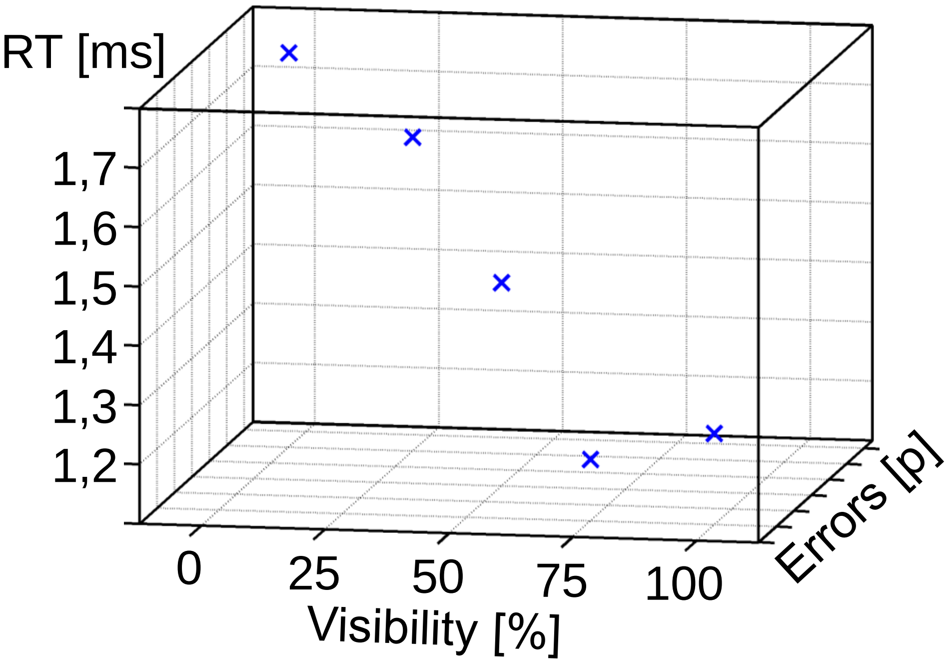


# Model optimization

- Add further regressors x
- Example: average error rate per level of visibility

| y | $x_1$ | $x_2$ |
|---|---|---|
| 1,77 | 0 | 0,5 |
| 1,63 | 25 | 0,5 |
| 1,44 | 50 | 0,3 |
| 1,20 | 75 | 0,1 |
| 1,25 | 100 | 0,1 |




$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_0 1 + \varepsilon$$

# Model optimization

Visibility $x_1$    Error rate $x_2$

Reaction times

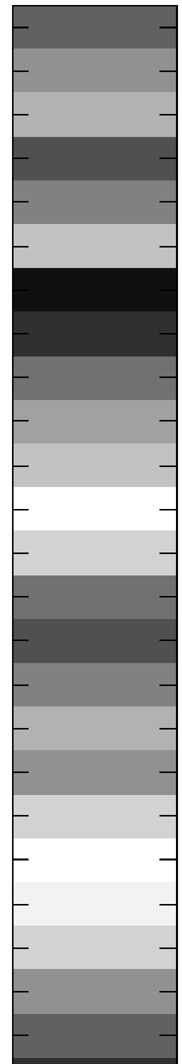
$$\begin{bmatrix} 1,765 \\ 1,629 \\ 1,443 \\ 1,204 \\ 1,254 \end{bmatrix} = \begin{bmatrix} 0 & 0,5 & 1 \\ 25 & 0,5 & 1 \\ 50 & 0,3 & 1 \\ 75 & 0,1 & 1 \\ 100 & 0,1 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_i \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

## fMRI

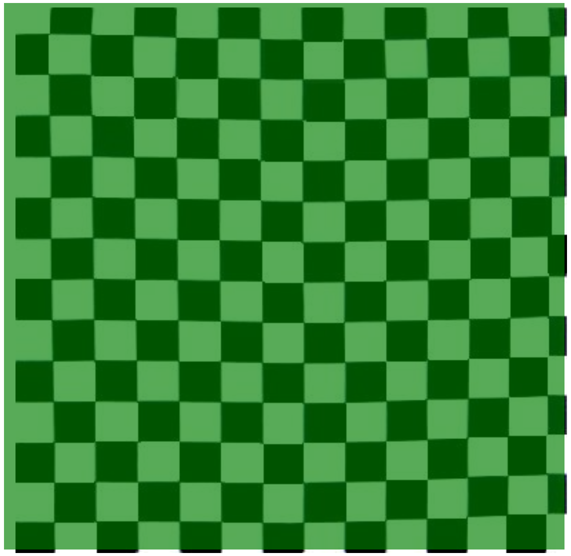
### y = BOLD signal intensity time course in a given voxel

Time course

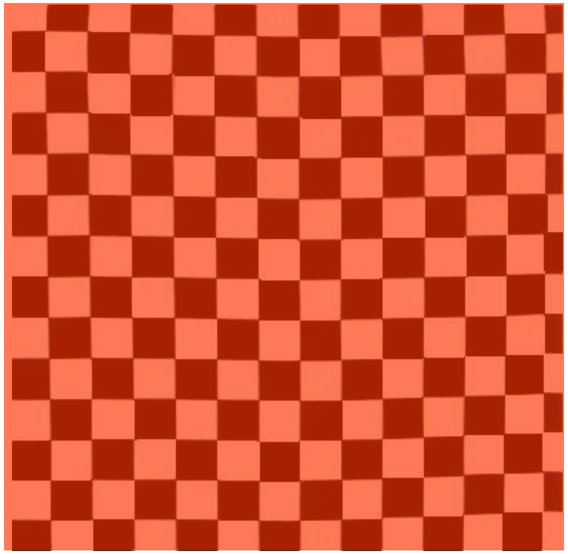
| Volume (Scan) | Signal intensity |
|---|---|
| 1 | 0.2 |
| 2 | 0.5 |
| 3 | 0.7 |
| 4 | 0.1 |
| 5 | 0.4 |
| 6 | 0.8 |
| 7 | -0.3 |
| 8 | -0.1 |
| 9 | 0.3 |
| 10 | 0.6 |
| 11 | 0.8 |
| 12 | 1.2 |
| 13 | 0.9 |
| 14 | 0.3 |
| 15 | 0.1 |
| 16 | 0.4 |
| 17 | 0.7 |
| 18 | 0.5 |
| 19 | 0.9 |
| 20 | 1.2 |
| 21 | 1.1 |
| 22 | 0.9 |
| 23 | 0.5 |
| 24 | 0.2 |



# fMRI



Rest



Finger tapping

Rest

**Hypotheses:**
i) Checkerboards vs. ITI → visual cortex
ii) Red vs. green checkerboard → motor cortex

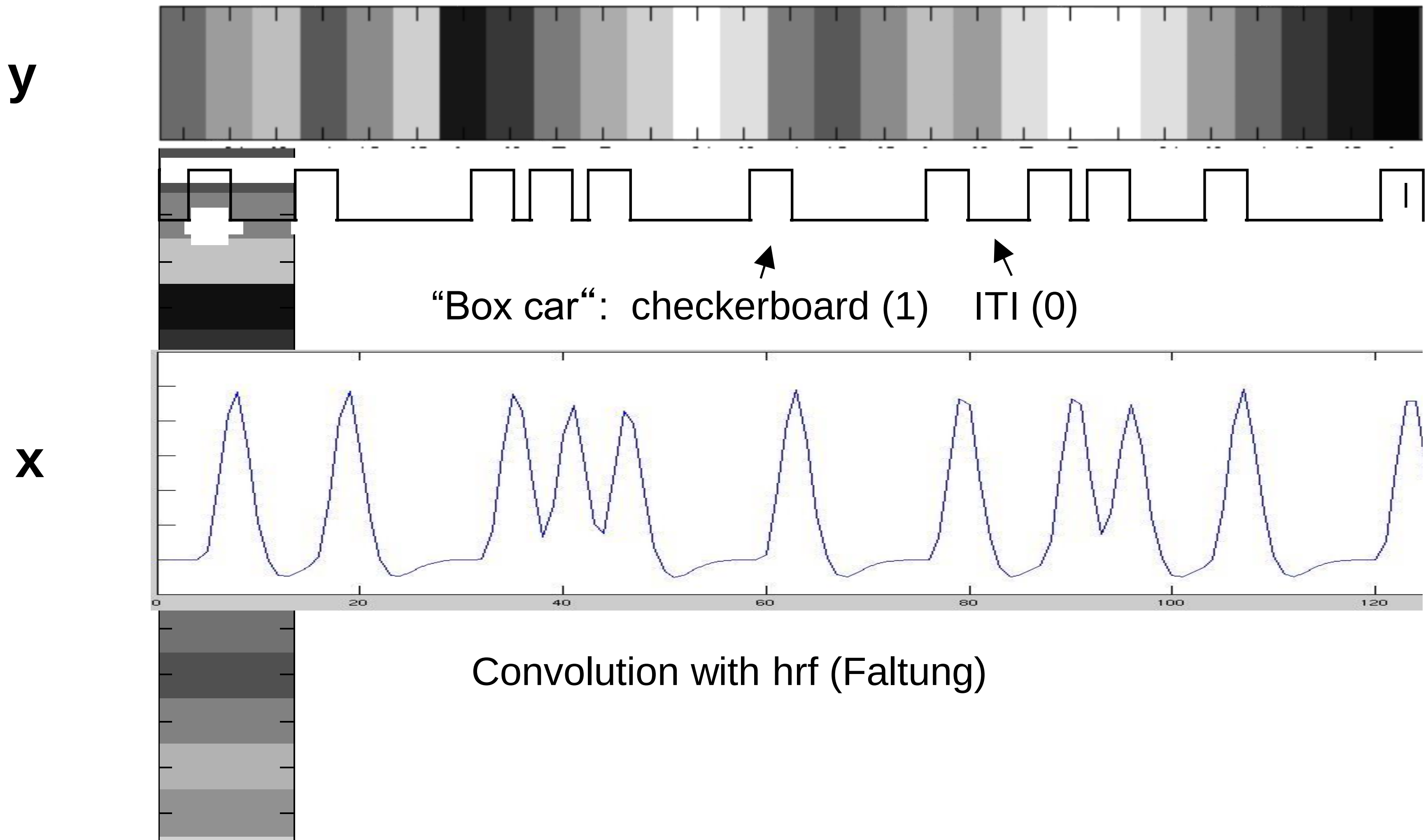
*Programming:*
Mareike Menz, ISN, UKE

# fMRI – choosing a model

**Hypotheses:**

i) Checkerboards vs. ITI → visual cortex

ii) Red vs. green checkerboard → motor cortex



# Choosing a model – example: linear relationship

$$y = X\beta + \varepsilon$$

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$x_1$ : Visibility

Reaction times

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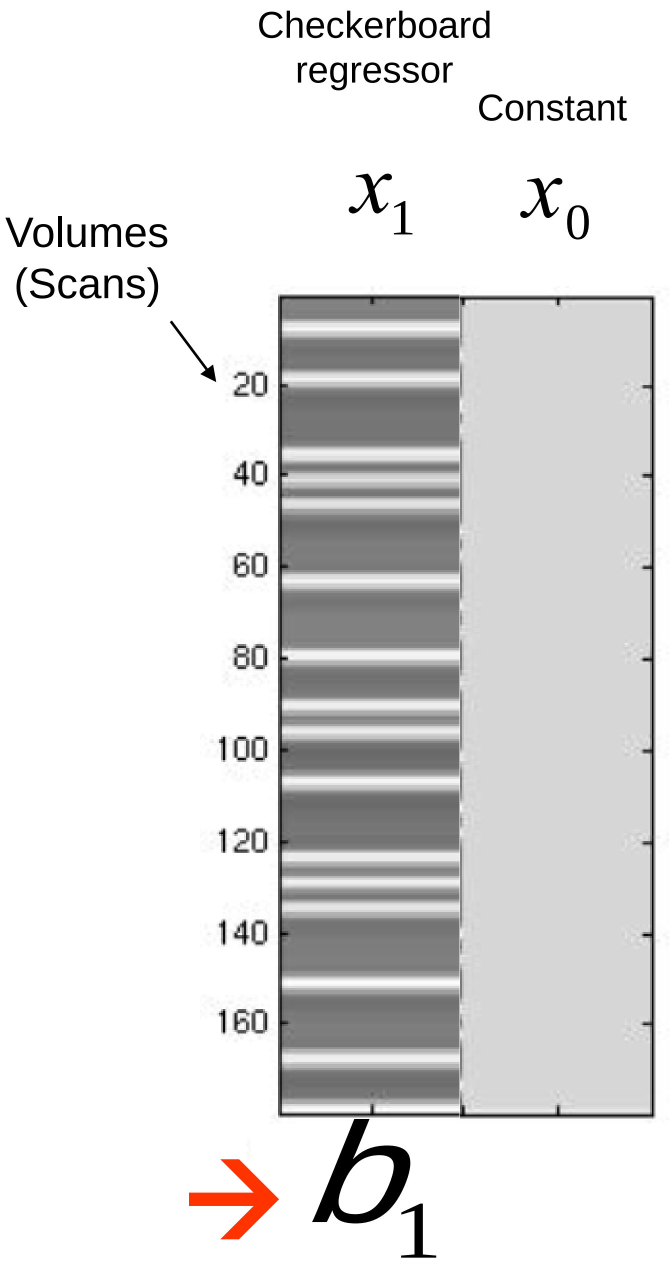
$\beta_0$ : “Y-Achsenabschnitt”

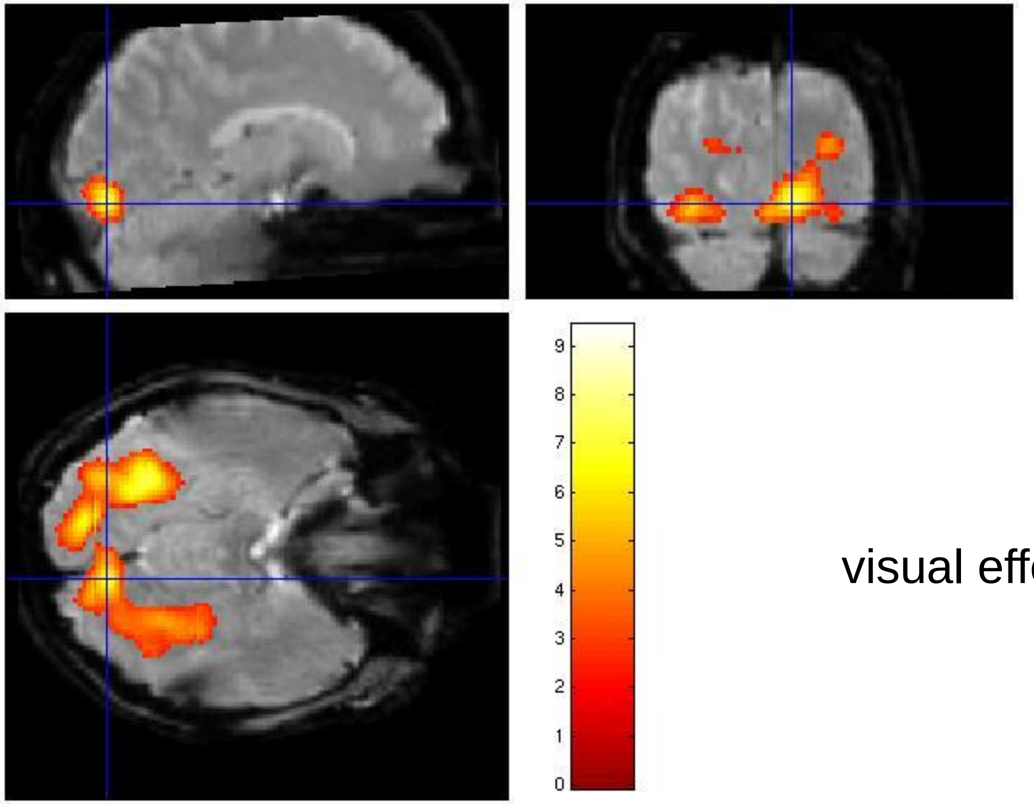
$\beta_1$ : Slope (Steigung)

# fMRI – choosing a model



# fMRI – parameter estimate maps (beta images)

$b_1$

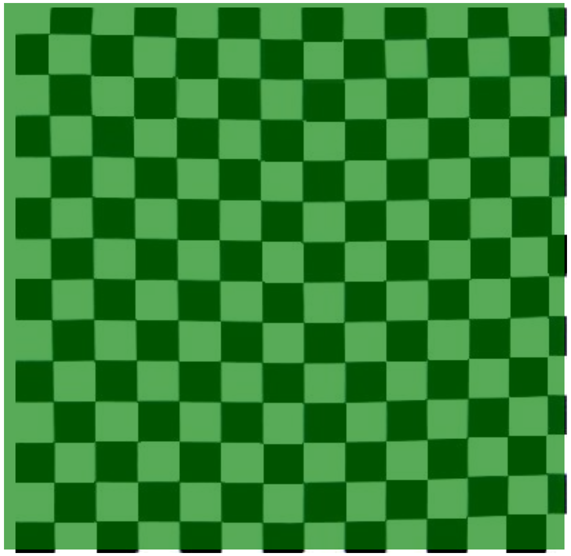



visual effect

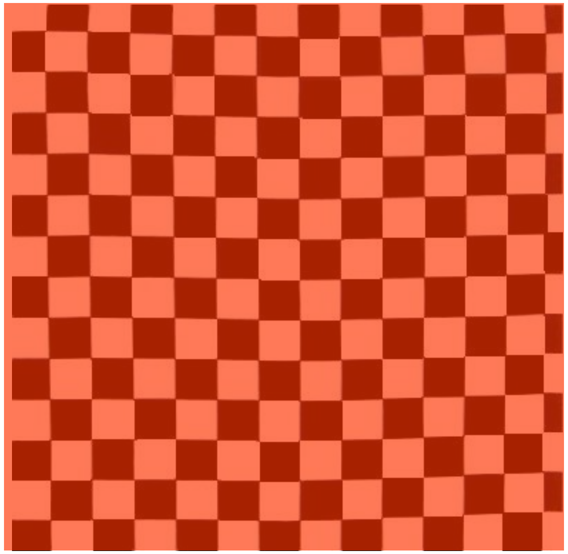
**⇒ separate GLM in EVERY voxel ("mass univariate approach")**

*Brain:*
Christian Paret, now ZI Mannheim

# fMRI



Rest



Finger tapping

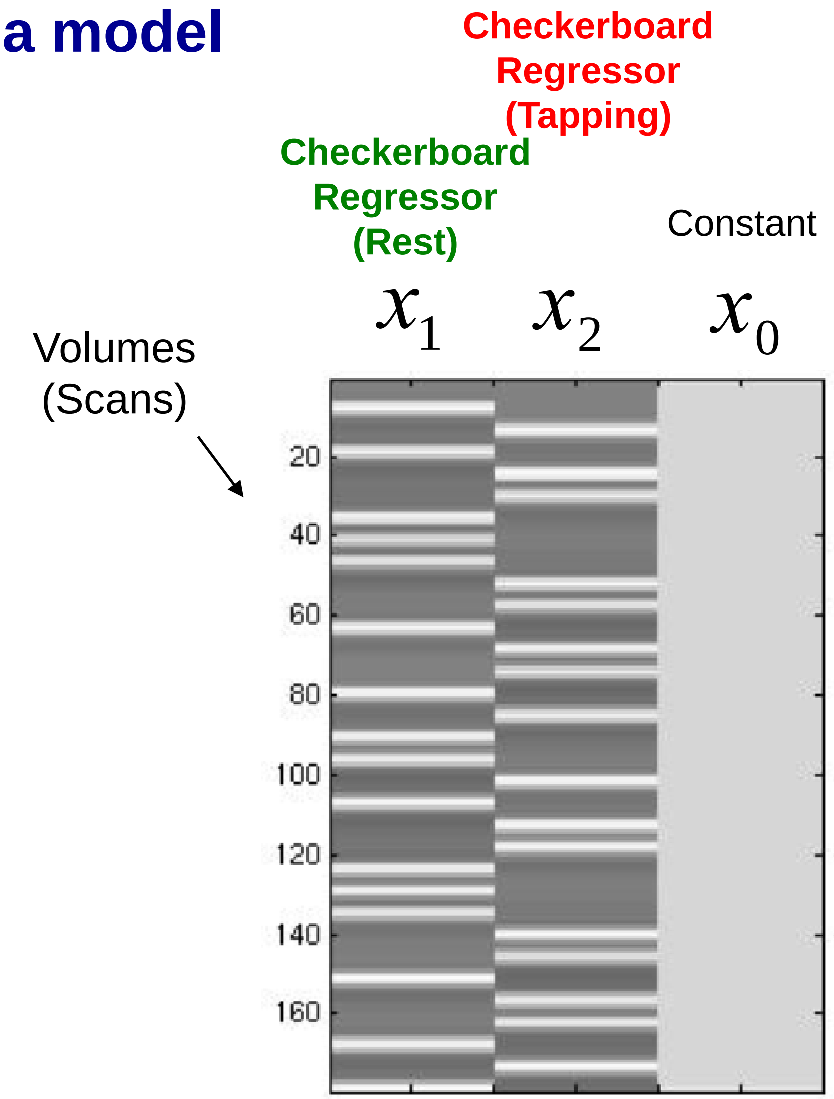
Rest

**Hypotheses:**

i) Checkerboards vs. ITI → visual cortex

ii) Red vs. green checkerboard → motor cortex

# fMRI – choosing a model




$\rightarrow b_1 \; b_2$

# Model optimization

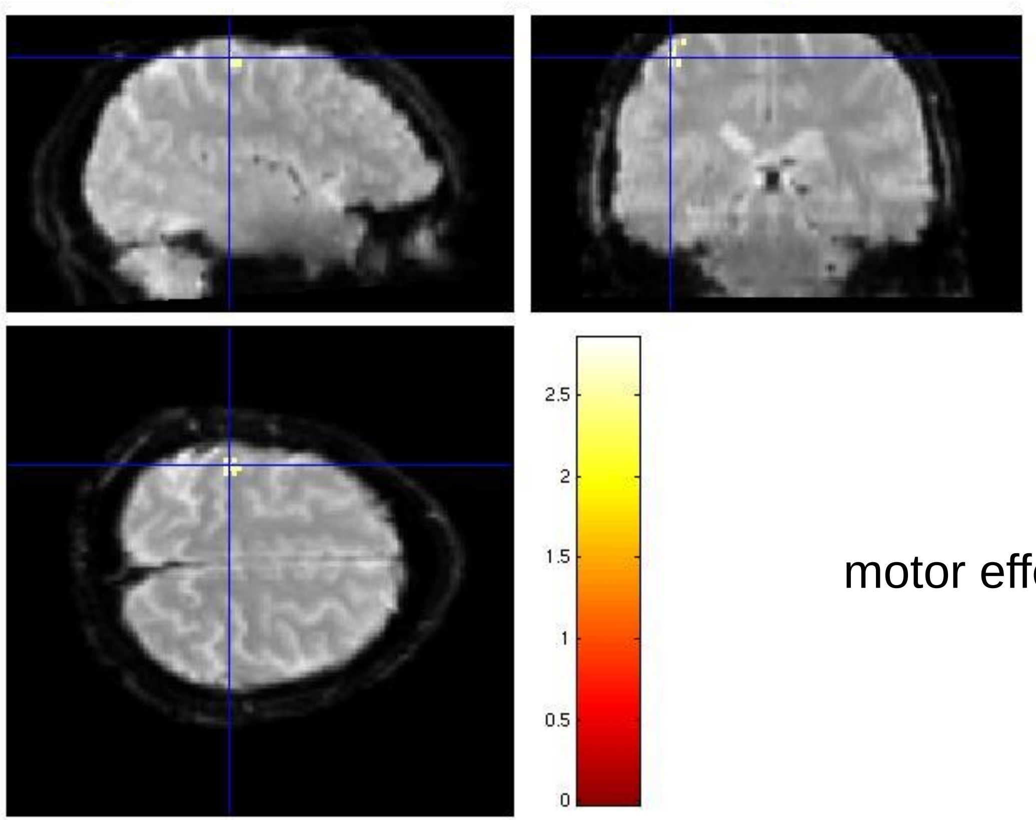
Visibility $x_1$ Error rate $x_2$

Reaction times

$$\begin{bmatrix} 1,765 \\ 1,629 \\ 1,443 \\ 1,204 \\ 1,254 \end{bmatrix} = \begin{bmatrix} 0 & 0,5 & 1 \\ 25 & 0,5 & 1 \\ 50 & 0,3 & 1 \\ 75 & 0,1 & 1 \\ 100 & 0,1 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_i \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

# fMRI – parameter estimate maps

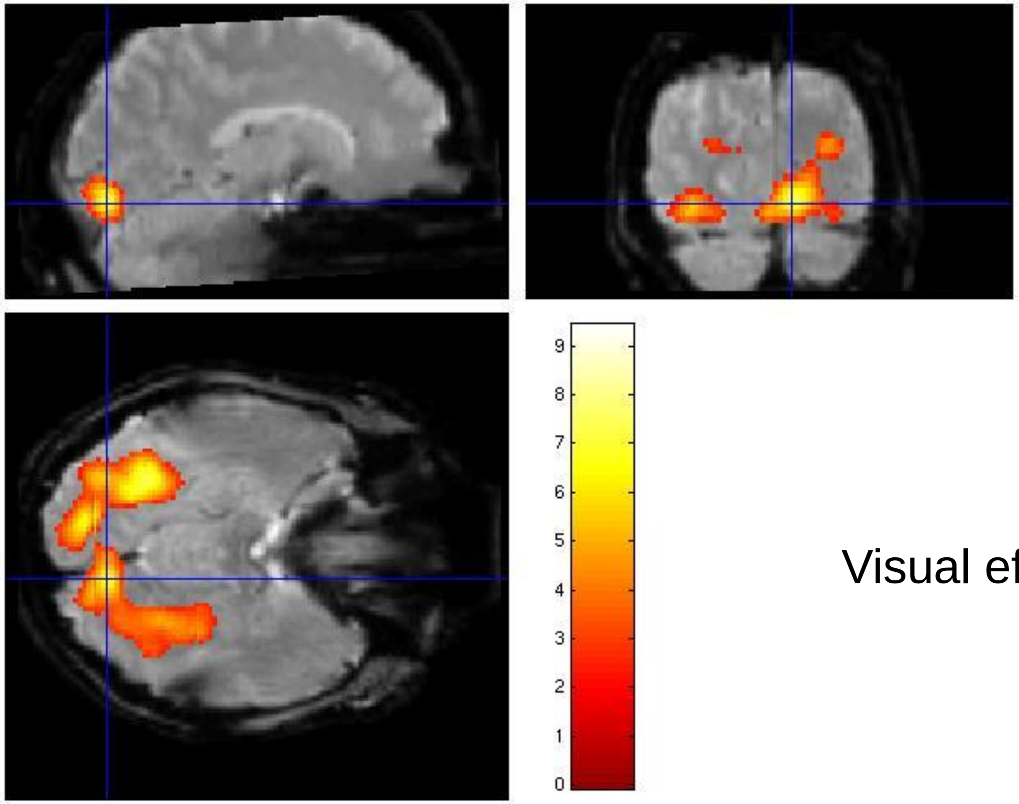
$$b_2 - b_1$$




motor effect

# fMRI – parameter estimate maps

$$\frac{b_2 + b_1}{2}$$

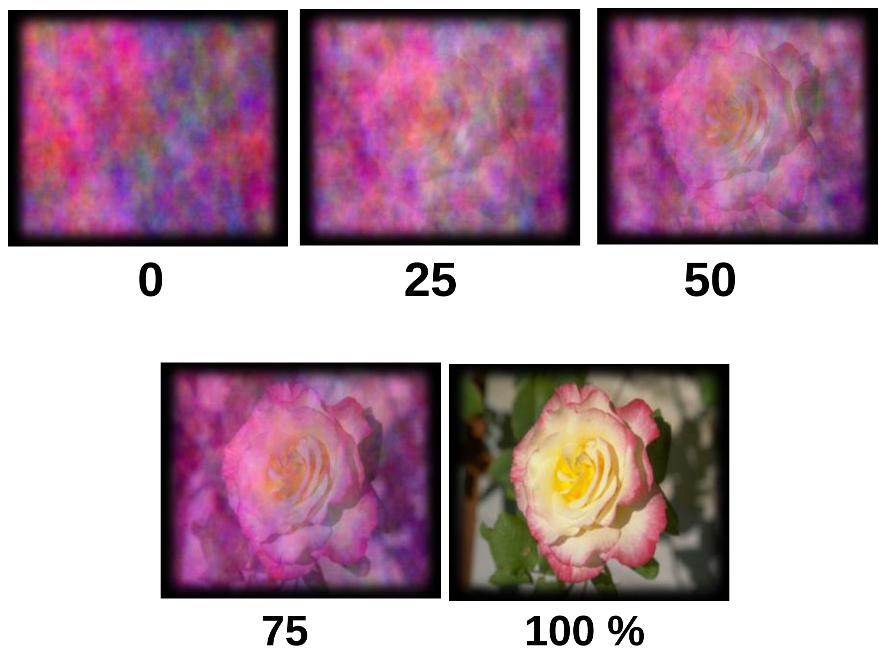



Visual effect

⇒ **calculate effects through linear combinations of beta images ("contrasts") → con images**

## fMRI – example study

- Implicit encoding of pictures (5 visibility levels)
- Crossed with cognitive task (n-back, 2 difficulty levels)
- Subsequent memory test: old/new decision



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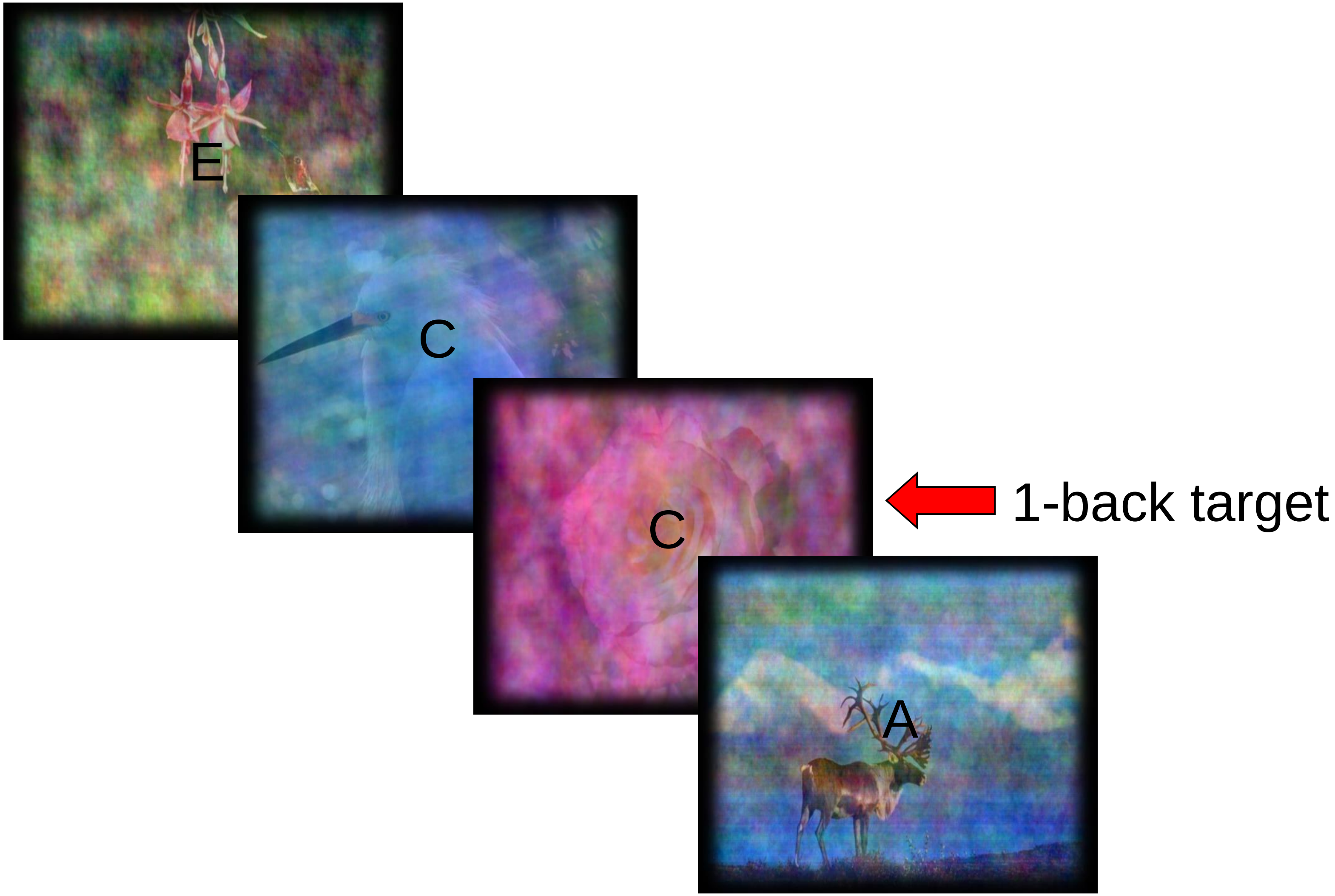
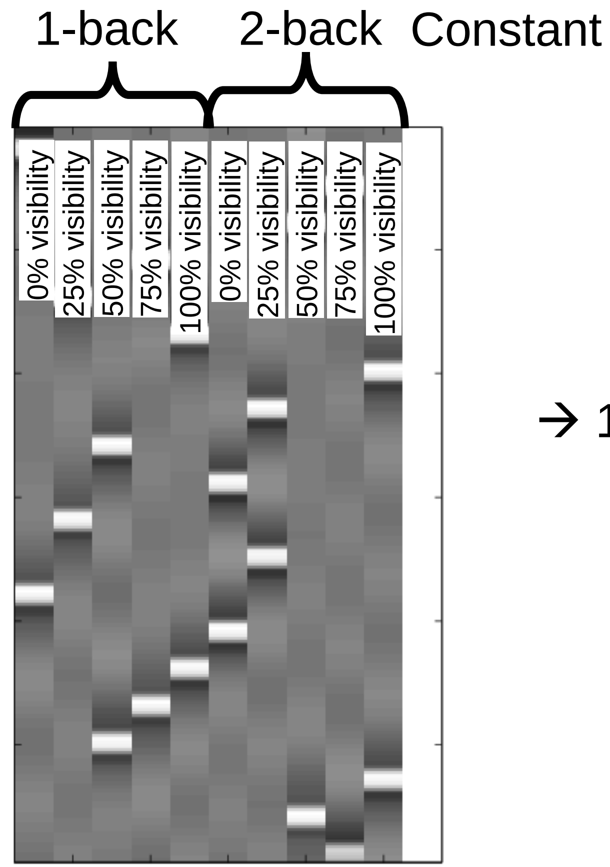


# fMRI – example study




→ 10 beta maps

$$b_1 \ldots b_{10}$$

**Contrasts:**

Visibility -2 -1 0 1 2 -2 -1 0 1 2

→ various con maps

## Summary

- GLM: (multiple) regression $y = ax + b + Error$
- dependent variable y, independent variable (regressor) x, error $e$
- regression weight (parameter estimate) $b$

$$y = X\beta + \varepsilon$$